Observation of negative refraction and negative phase velocity in left-handed metamaterials

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We report the transmission characteristics of a two-dimensional (2D) composite metamaterial (CMM) structure in free space. At the frequencies where left-handed transmission takes place, we experimentally confirmed that the CMM structure has effective negative refractive index. Phase shift between consecutive numbers of layers of CMM is measured and phase velocity is shown to be negative at the relevant frequency range. Refractive index values obtained from the refraction experiments and the phase measurements are in good agreement. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1888051]

In 1968, Veselago predicted that a medium with negative permittivity, ε, and negative permeability, μ, will exhibit unusual physical properties like negative refraction, reversal of Doppler shift, and backward Cherenkov radiation.[1] In such a medium, the electric, magnetic, and wave vector components form a left-handed (LH) coordinate system, hence the name left-handed material (LHM) is used for description. Recently, this idea is brought to experimental investigation by constructing a composite metamaterial (CMM) consisting of two components which have $\varepsilon(\omega)<0$ and $\mu(\omega)<0$ simultaneously over a certain frequency range.[2–4] These components are realized as periodically arranged metallic wires,[5] and split ring resonators (SRR).[6] In these experiments, the conditions for LHM behavior of the CMM is stated intuitively as follows: Around the magnetic resonance frequency of SRRs ($\omega \sim \omega_m$), both the SRR-only medium [having $\mu_{\mathrm{SRR}}(\omega)<0$ and $\varepsilon_{\mathrm{SRR}}(\omega)>0$] and the wire-only medium [having $\varepsilon_{\mathrm{wire}}(\omega)<0$ and $\mu_{\mathrm{wire}}(\omega)=1$] will have stop bands, provided that $\omega_m<\omega_p$, where ω_p is the plasma frequency of wire medium. The composite medium will then have $\varepsilon(\omega)<0$ and $\mu(\omega)<0$, and consequently a pass band, which should act as a LHM. However, we have recently shown that the SRR-only medium also has dielectric response, which reduces the cutoff frequency of the composite medium significantly from that of the wire-only medium.[7–9] Therefore, the condition for identifying a LHM pass band unambiguously is that $\omega_m(\mathrm{SRR})<\omega_p(\mathrm{CMM})$.

Negative refraction in wedge structures is the typical experimental method used for observation of left-handed properties in CMMs.[10–12] However, reversal of phase velocity can also be used as an indication of LH behavior.[12–14] In this letter, we present direct experimental evidence that both the phase velocity and the refractive index is negative within the LH pass band of a CMM.

The SRR and wire patterns are fabricated on the front and back sides of FR4 circuit boards which have 30 μm thick deposited copper layer. The geometrical parameters of a single SRR unit can be found in our previous work.[7] The length and width of the wire structures are $l=19$ cm, and $w=0.9$ mm, respectively. The unit cell consists of two SRRs and two wires in x-z planes, as shown in shaded parts of Fig. 1(a). The 2D CMM structure is made of $N_x=5$, $N_y=20$, and $N_z=40$ unit cells, with lattice spacings $a_x=a_y=a_z=9.3$ mm. Transmission measurements are performed in free space. Experimental measurement setup consists of an HP 8510C network analyzer, and a set of microwave horn antennas. The incident electromagnetic (EM) wave propagates along the x direction, while **E** is along y direction, and **H** is along the z direction [Fig. 1(a)].

Figure 2 shows the measured transmission spectra of periodic SRRs (solid line) and wires (dashed line) and 2D

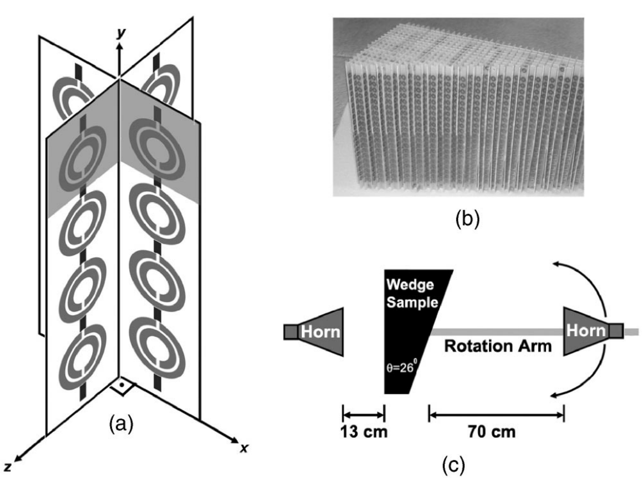

FIG. 1. (a) Schematics of 2D CMM structure; (b) 2D wedge CMM structure used for negative refraction experiment; (c) schematic drawing of experimental setup used for refraction experiment.

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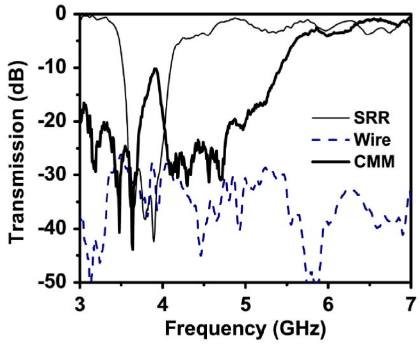

FIG. 2. Measured transmission spectra of a periodic SRR medium (solid line), periodic wire medium (dashed line) and 2D CMM medium (bold solid line) between 3 and 7 GHz.

CMM between 3 and 7 GHz. The band gap of SRR between 3.55 and 4.05 GHz is due to magnetic resonance of periodic SRR medium, hence $\mu(\omega)<0$ for this frequency range.[7] The 2D CMM structure allows propagation of EM waves between 3.7 and 4.1 GHz, where both ε and μ are negative. The CMM pass band coincides with the stop band of SRR. The transmission peak at 3.92 GHz is -10.2 dB, which is significantly higher than the previously reported 2D CMM structures.[3,11] We also reported high transmission (-1.2 dB) for 1D CMM structures with the same parameters[7] so it is not surprising to obtain high transmission for the 2D case. The high transmission can be explained by better impedance matching between air and CMM for this particular CMM design. The transmission band starting from 5.3 GHz is due to downward plasma frequency shift, since the $\varepsilon>0$ regime of the combined electric response of SRRs and wires starts at 5.3 GHz.[7]

For negative refraction experiments, a prism shaped 2D CMM structure is constructed [Fig. 1(b)]. The minimum and maximum number of unit cells at the propagation direction is 3 and 19, which results in a wedge angle of $\theta=26°$. Figure 1(c) depicts the schematic of the experimental setup. The source is 13 cm ($\sim 2\lambda$) away from the first interface of the wedge. Full width half maximum of the beam (9.5 cm) at the first interface is smaller than the size of the incident surface (34 cm). Receiver antenna is mounted on a rotating arm to obtain the angular distribution of the transmitted signal. Receiver antenna is located at a distance of 70 cm ($\sim 10\lambda$) away from the second interface of the wedge.

The angular refraction spectrum is scanned by $\Delta\theta=2.5°$ steps, while the frequency is swept from 3.73 to 4.05 GHz. Figure 3(a) displays the transmission spectrum as a function of frequency and refraction angle. It is evident from the figure that the transmitted beam is refracted on the negative side of the normal. In contrast to the previous negative refraction experiments,[10–12] the refraction index is measured to be negative for the entire LH transmission band. At lower frequencies the EM waves are refracted at higher negative refraction angles, which results in a higher negative refractive index. The refraction index is lowered if we go to higher frequencies.

To investigate the beam profile, the angular cross section at $f=3.92$ GHz is plotted in Fig. 3(b). By employing Snell's law ($n_{\mathrm{CMM}} \sin\theta_i = n_{\mathrm{air}} \sin\theta_r$) an effective refractive index can be defined for the CMM. For $\theta_i=26°$, EM wave is refracted at an angle of $\theta_r=55°$, then from Snell's law we obtain $n_{\mathrm{eff}}=-1.87\pm0.05$ at 3.92 GHz.

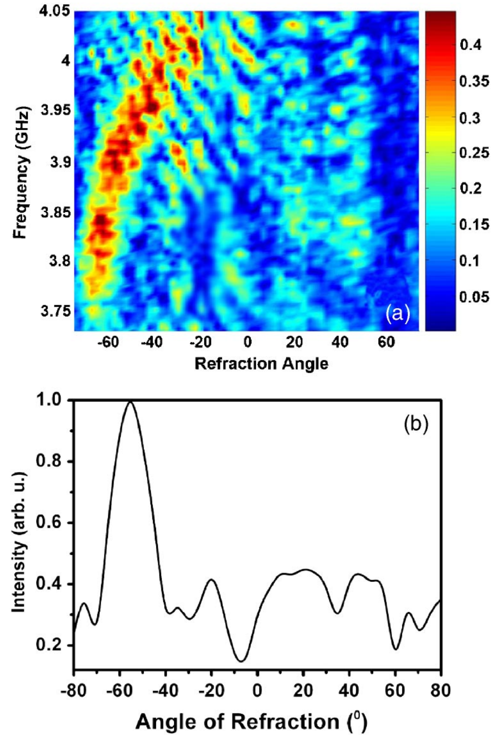

FIG. 3. (a) (Color online)Transmission spectra as a function of frequency and refraction angle (b) The angular cross section of transmitted beam at $f=3.92$ GHz.

The transmitted phase of CMMs is measured to investigate the phase velocity within both the left-handed (3.73–4.05 GHz) and right-handed (5.4–7.0 GHz) transmission bands (Fig. 2). Phase measurements are performed on rectangular slabs of CMMs, with various numbers of layers. Figure 4 shows the transmitted phase of CMMs (with varying number of layers) between frequencies 5.4 and 7.0 GHz, where CMM acts as a right-handed medium. As shown in Fig. 4, the phase of the transmitted EM wave increases, when a longer CMM is used, which is a typical right-handed behavior. On the other hand, increasing the number of layers decreases the phase of the transmitted EM wave at the left-handed frequency region (Fig. 5). As shown in the inset of Fig. 5, the average phase shift is negative for the relevant frequency range, which indicates that the phase velocity is negative.

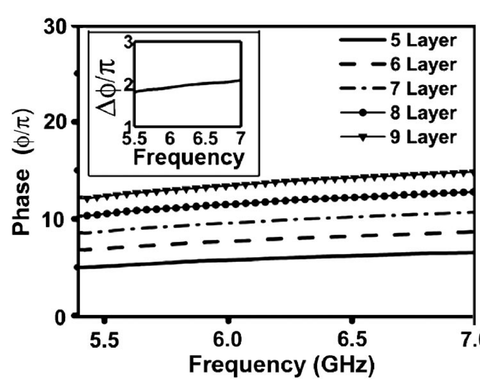

FIG. 4. Unwrapped transmission phase data obtained from different lengths of CMM between 5.4 and 7.0 GHz, where right-handed transmission peak takes place. Inset: Average phase difference between consecutive numbers of layers of CMM. Phase shift is positive between 5.4 and 7.0 GHz.

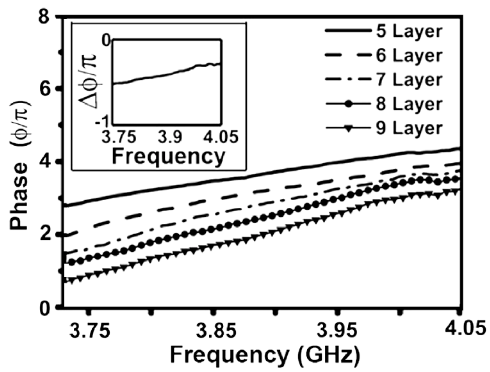

FIG. 5. Unwrapped transmission phase data obtained from different lengths of CMM between 3.73 and 4.05 GHz, where left-handed transmission peak takes place. Inset: Average phase difference between consecutive numbers of layers of CMM. Phase shift is negative between 3.73 and 4.05 GHz.

One can find the value of refractive index by using the phase shift between consecutive numbers of layers of CMM. Phase velocity is defined as $v_{ph}=c/n$, and also given by $v_{ph}=\omega/k$. Then, refraction index can be defined as $n=k.c/\omega$, where $k=\Delta\Phi/\Delta L$. We then obtain the refraction index as:

$$n=\frac{\Delta\phi}{\Delta L}\cdot\frac{c}{\omega}. \tag{1}$$

At $f=3.92$ GHz, the average phase shift between CMM layers is $\Delta\Phi=-0.41\pm0.05\pi$. By employing Eq. (1), n_{eff} is obtained to be -1.78 ± 0.22, which is in good agreement with the value of -1.87 ± 0.05 obtained from the refraction experiment. [The errors for index of refraction values obtained from refraction experiment is due to the finite angular step size ($\Delta\theta=25°$). The errors for phase shift experiment is a result of the measured phase differences ($\pm0.05\pi$) between consecutive number of layers.] For $f=3.84$ GHz, n_{eff} obtained from the refraction experiment is found to be $n=-2.02\pm0.05$, while the phase shift experiment gives $n=-1.97\pm0.22$. Also at $f=3.98$ GHz, the refraction experiment results ($n=-1.22\pm0.05$), and the phase shift experiment results ($n=-1.28\pm0.21$) are in good agreement. The measured phase velocity at 3.92 GHz is negative and equal to $-0.51\ c$.

In conclusion, we have demonstrated a left-handed transmission band for 2D CMM structure in free space with a high transmission peak. We experimentally confirmed that 2D CMM has negative refractive index at the entire left-handed frequency range (3.73–4.05 GHz). Phase shift and therefore phase velocity is shown to be negative, and the values of negative refractive indices obtained from the refraction experiments and the phase measurements are in good agreement.

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